Domain wall dynamics in soft magnetic materials

Alexei Vázquez[1] and Oscar Sotolongo-Costa[1,2]
[1] *Department of Theoretical Physics, Faculty of Physics, Havana University, Havana 10400, Cuba*
[2] *LICTDI, Faculty of Sciences, UNED, Madrid 28080, Spain*

The dynamics of a domain wall in magnetostrictive materials is investigated. The domain wall is modeled by a d-dimensional interface moving in a $d+1$-dimensional environment. Long-range demagnetization effects and quenched disorder are considered, while the external magnetic field is increased at constant rate. Exact expressions for the average interface velocity and susceptibility are obtained, resulting that the system is critical when the demagnetization constant and the average interface velocity vanishes. The critical exponents are computed to $\mathcal{O}(\varepsilon=4-d)$. Our predictions are compared with numerical simulations and Barkhausen noise measurements reported in the literature.

68.35.Rh, 05.40.+j, 75.60.Ej, 64.60.Lx

Domain walls in ferromagnets move following an irregular motion in response to changes in an applied external magnetic field, leading to discrete jumps in the magnetization. This phenomenon, known as the Barkhausen effect, has received a renewed theoretical and experimental interest, because of its connection with non-equilibrium critical phenomena like interface depinning and self-organized criticality [1–5].

Urbach *et al* [1] have analyzed the role of demagnetization effects in the avalanche statistics. From numerical simulations and Barkhausen noise measurements, they conclude that the addition of an "infinite-range" demagnetization field results in self-organized criticality, while a local demagnetization field does not. On the other hand, Narayan [2] has shown that long-range dipolar interactions give rise to self-similar dynamics. More recently, Cizeau *et al* [3] derived an equation of motion for the dynamics of a ferromagnetic domain wall, including both long-range demagnetization effects and dipolar interactions. From numerical simulations, they conclude that long-range dipolar interactions decreases the upper critical dimension from $d_{uc}=4$ to $d_{uc}=2$ [6]. Moreover, they have also demonstrated that the demagnetization factor acts as a control parameter, criticality is obtained after it is fine tunned to zero.

In magnetostrictive materials magneto-elastic interactions enhance the surface tension effect over dipolar interactions and, therefore, scaling exponents for two-dimensional domain walls should be lower than the mean-field ones. Experimental measurements of Barkhausen noise with different magnetostrictive materials corroborate this conclusion [1,4,5].

In a recent work [7] we have analyzed the dynamics of a d-dimensional interface under a driving force increasing linearly with time, a restored force proportional to the local interface height $H(\vec{x},t)$, and quenched disorder (This model is equivalent to the Urbach *et al* model with local demagnetization effects). Extending previous renormalization group (RG) calculations developed for the constant force case [8,9] we obtained the dynamic z and roughness ζ exponents in first order of $\varepsilon=4-d$. Here we use a similar approach to analyze the same model but with long-range demagnetization effects. The starting equation of motion is a particular case of that obtained by Cizeau *et al* [3], excluding long-range dipolar interactions. It is demonstrated that the model with long-range demagnetization effects belongs to the same universality class as that of local demagnetization effects [7]. Our predictions are compared with numerical simulations and Barkhausen noise measurements, reported in the literature.

In the central part of the hysteresis loop, around the coercitive field, the magnetization process is mainly due to domain wall motion, while domain nucleation and rotation are absent. Moreover in most ferromagnetic materials, due to magnetocrystalline forces or sample shape, the magnetization has preferred directions. In the simplest situation, there is a single easy axis of magnetization and the domains are separated by surfaces, parallel to the magnetization and spanning the sample from end to end. In this situation the domain wall dynamics may be modeled as the motion of a d-dimensional interface in a $d+1$ environment, with height $H(\vec{x},t)$. In magnetostrictive materials the long range dipolar interactions can be neglected, resulting the equation of motion [3]

$$\lambda\frac{\partial}{\partial t}H(\vec{x},t)=\Gamma\nabla^2H(\vec{x},t)+ht-\epsilon\int\frac{d^dx'}{L^d}H(\vec{x}',t)+\eta[\vec{x},H(\vec{x},t)], \tag{1}$$

where Γ is the surface tension of the wall, λ represents a friction coefficient, ht is the applied field which is supposed to increase at rate h, L is the linear size of the system, and ϵ is the demagnetization factor. Here $\eta(\vec{x}, H)$ is a Gaussian uncorrelated noise due to impurities, with zero mean and

$$\langle \eta(\vec{x}, H)\eta(\vec{x}', H')\rangle = \delta^d(\vec{x} - \vec{x}')\Delta(H - H'), \tag{2}$$

where $\Delta(H)$ is a monotonically decreasing even function, with a fast decay to zero beyond a distance $a_\perp$. In eq. (1) we have explicitly taken into account that the demagnetization constant scales as the inverse of the system size, which for an hyper-cubic lattice scales as L^d. Hence, the demagnetization factor ϵ does not depend on system size. Eq. (1) is similar to Edwards-Wilkinson equation with quenched noise, but the external field increases at rate h and there is an additional term due to demagnetization effects. This difference carry as a consequence that the interface is never pinned by impurities, but always moves with a finite average velocity v.

A perturbative solution of eq. (1) can thus be found expanding $H(\vec{x}, t)$ around the flat co-moving interface vt. Taking $H(\vec{x}, t) = vt + w(\vec{x}, t)$, with $\langle w(\vec{x}, t)\rangle = 0$, we obtain the following equation for $w(\vec{x}, t)$

$$\lambda \frac{\partial}{\partial t} w(\vec{x}, t) = \Gamma \nabla^2 w(\vec{x}, t) + (h - \epsilon v)t - \lambda v - \epsilon \int \frac{d^d x'}{L^d} w(\vec{x}', t) + \eta[\vec{x}, vt + w(\vec{x}, t)]. \tag{3}$$

The average velocity is obtained using the constraint $\langle w(\vec{x}, t)\rangle = 0$, while fluctuations around the average will be analyzed using RG transformations. For these purposes is better to work with the equation for the Fourier transform of $w(\vec{x}, t)$, $\tilde{w}(\vec{k},\omega)$ [9]. The external field $(h - \epsilon v)t$ gives a singular term of the order of ω^{-2}. This singular term predominates in the low frequency limit resulting, after imposing $\langle \tilde{w}(\vec{k}, \omega)\rangle = 0$,

$$v = \frac{h}{\epsilon}. \tag{4}$$

Another exact result can be obtained if one computes the low-frequency and long-wavelength susceptibility. Adding a source term $\hat{\varphi}(\vec{k}, \omega)$ to the equation for $\tilde{w}(\vec{k},\omega)$ one obtains the generalized response function [9]

$$\tilde{G}(\vec{k}, \omega) = \left.\frac{\tilde{w}(\vec{k}, \omega)}{\tilde{\varphi}(\vec{k}, \omega)}\right|_{\tilde{\varphi}=0} = \frac{1}{[\tilde{G}_0(\vec{k}, \omega)]^{-1} - \tilde{\Sigma}(\vec{k}, \omega)}, \tag{5}$$

where

$$[\tilde{G}_0(\vec{k}, \omega)]^{-1} = \Gamma \vec{k}^2 + i\lambda\omega + \epsilon\hat{\delta}(\vec{k}, L). \tag{6}$$

is the bare correlator and $\tilde{\Sigma}(k, \omega)$ is the "self-energy". $\hat{\delta}(\vec{k}, L)$ is the Fourier transform of L^{-d}. In the thermodynamic limit $(L \to \infty)$ $\hat{\delta}(\vec{k}, L) \approx 1$ for $\vec{k} \to 0$ and zero otherwise. Since $\tilde{\Sigma}(0, 0) = 0$ and $\tilde{G}_0(0, 0) = \epsilon^{-1}$ it results that the low-frequency and long-wavelength susceptibility (or simply the susceptibility) is given by

$$\chi = \tilde{G}(0, 0) = \frac{1}{\epsilon}. \tag{7}$$

This result is also exact.

In the limit $\epsilon \to 0$ the susceptibility diverges as $\chi \sim \epsilon^{-\gamma}$ with

$$\gamma = 1. \tag{8}$$

The system is thus critical for $\epsilon \to 0$, i.e. when demagnetization effects becomes negligible. However, different behaviors can be observed depending on the value of h. If h is finite and we take $\epsilon \to 0$ then, according to eq. (4), the domain wall velocity will become infinitely large and, therefore, the noise term in eq. (3) will be reduced to an annealed noise. Thus, for ($h > 0$, $\epsilon \to 0$) the interface dynamics is described by the EW equation with annealed noise, which has an upper critical dimension 2. Hence, for $d \geq 2$ the interface will moves with a velocity given by eq. (4) and fluctuations around the average can be neglected. This picture corresponds with a suppercritical regime, where the external magnetic field predominates over disorder. On the contrary, for ($h \to 0$, $\epsilon > 0$) the system will be in a subcritical regime. The domain wall will moves with an infinitely small velocity and, therefore, the quenched noise will lead to fluctuations around the flat co-moving interface, but these fluctuations will be not critical due to demagnetization effects. This fact becomes evident with the existence of a finite susceptibility. Finally, the critical state corresponds with the double limit $\epsilon \to 0$ and $v = h/\epsilon \to 0$.

To go further we perform a RG analysis of the problem. We integrate out the degrees of freedom in a momentum shell near the cutoff Λ and rescale $k \to b^{-1}k$, $\omega \to b^{-z}\omega$, and $\tilde{w} \to b^{\zeta+d+z}\tilde{w}$, where $b = \mathrm{e}^l$ with $l \to 0$. The flow equations for the parameters Γ, λ, ϵ and v are obtained through a direct application of the RG transformations to eq. (3) in the Fourier space resulting, to one-loop order,

$$\frac{d\Gamma}{dl} = 0, \qquad \frac{d\lambda}{dl} = \lambda(2 - z + cQ_2), \tag{9}$$

$$\frac{d\epsilon}{dl} = 2\epsilon, \qquad \frac{dv}{dl} = (z - \zeta)v. \tag{10}$$

where $c = \frac{S_d}{(2\pi)^d \Gamma^2}\Lambda^{-\varepsilon}$, S_d is the complete solid angle in d dimensions, and $Q_2 = \int_q \tilde{\Delta}(q)q^2$ is the second moment of the noise correlator. Here we have implicitly assumed that $\Gamma\Lambda^2 \gg \epsilon$ and $\Gamma\Lambda^2 \gg \lambda v/a_\perp$, as it is expected near the critical point where both ϵ and v should vanishes. The long-range nature of demagnetization effects is, within our RG approach, irrelevant. The same result will be obtained if one considers a local demagnetization term $-\epsilon H$. Hence, all the analysis developed below will also hold for local demagnetization effects.

The renormalization of the moments of the noise correlator ($Q_n = \int_q \tilde{\Delta}(q)q^n$) is obtained considering a vertex function [9,8]. The renormalization equations for the moments can be put all together in a renormalization equation for the noise correlator $\Delta(H)$, obtaining

$$\frac{\partial \Delta(H)}{\partial l} = (\varepsilon - 2\zeta)\Delta(H) + \zeta H \Delta'(H) - c\frac{d^2}{dH^2}\left[\frac{1}{2}\Delta^2(H) - \Delta(H)\Delta(0)\right]. \tag{11}$$

We look for a fixed point solution $\Delta^*(H)$ to this equation, obtained setting $\partial\Delta/\partial l$ to zero and adjusting ζ. Integrating from $-\infty$ to ∞ one obtains [8,9]

$$\zeta = \frac{\varepsilon}{3}, \tag{12}$$

provided $\int_{-\infty}^{\infty} dH \Delta^*(H) \neq 0$. On the other hand, for $H \to 0$ we obtain $\Delta^*(H) \approx Q_0 + Q_1|H| - \frac{1}{2}Q_2 H^2$ with

$$cQ_2 = -\frac{\varepsilon - \zeta}{3}. \tag{13}$$

Then, imposing scale invariance in eq. (10) and using eqs. (12) and (13) it results that

$$z = 2 - \frac{2}{9}\varepsilon. \tag{14}$$

The exponents ζ and z thus result identical to those obtained for the constant force case [8,9].

On the other hand, eq. (10) implies that there is a characteristic lenght $\xi \sim \epsilon^{-\nu}$ beyond which scale invariance is lost due to demagnatization effects, and a characteristic velocity $v_c \sim \epsilon^{\theta}$ (or field rate $h_c = \epsilon v_c$) beyond which the system is supercritical, where

$$\nu = \frac{1}{2}, \qquad \theta = \nu(z - \zeta). \tag{15}$$

The line $v_c \sim \epsilon^{\theta}$ devides the phase diagram (v, ϵ) into two regions. For $v > v_c$ the noise is annealed and, therefore, the model is supercritical as discussed above. On the contrary, for $v < v_c$ the model is subcritical for all $\epsilon > 0$. The critical state is obtained in the double limit $\epsilon \to 0$ and $v = h/\epsilon \to 0$. The scaling exponents $\gamma = 1$ and $\nu = 1/2$ are exact while ζ and z are given by eqs. (12) and (14), respectively, up to the first order in ε.

In the subcritical regime the dynamics takes place in the form of avalanches, characterized by the avalanche size $P(s) = s^{-\tau} f(s/s_c)$ and duration $P(T) = T^{-\alpha} g(T/T_c)$ distributions, where s_c and T_c are the avalanche size and duration cutoffs. In the subcritical state $s_c \sim \epsilon^{-1/\sigma}$. At criticality $s_c \sim L^D$ and $\xi \sim L$, where D is the avalanche dimension, leading to the scaling relation $\sigma = 1/D\nu$. Other scaling relations are obtained taking into account that $\chi = \langle s \rangle$ and $\int ds P(s) = \int dT P(T)$, which lead to $\gamma = (2-\tau)/\sigma$ and $(\tau - 1)D = (\alpha - 1)z$, respectively. Taking γ, ν and D as independent exponents we obtain the scaling relations for the avalanche exponents

$$\tau = 2 - \frac{\gamma}{D\nu}, \qquad \alpha = 1 + \frac{D - \gamma\nu^{-1}}{z}. \tag{16}$$

On the other hand, for $d < d_c$, the avalanche dimension and the roughness exponent are related via $D = d + \zeta$. Hence, we can compute τ and α using the RG estimates of ζ and z and the exact values of γ and ν. Above the upper critical dimension $d_{uc} = 4$, one obtain the mean-field exponents $D = d_{uc} = 4$, $\tau = 3/2$ and $\alpha = 2$.

Before compare our exponent predictions with experiments and simulations, let us make some remarks about experimental measurements. The Barkhausen signal $V(t)$ is the voltage produced from a pickup coil around a ferromagnet subjected to a slowly varying applied field. In the low-frequency limit the time scale for domain wall motion is much smaller than the time between jumps and, therefore, one may guarantee that each induced voltage jump corresponds with a single avalanche in the domain wall motion. A resolution voltage level V_R is defined, such that one can not resolve details below V_R. An elementary Barkhausen jump can thus be defined as the portion of the $V(t)$ signal delimited by two subsequent intersections of the signal with the V_R line. With this definition, the duration T is simply the time interval between these two subsequent intersections and the size s is the area delimited by $V(t)$ and V_R between the same points.

Evaluating the scaling exponents for $d = 2$, using the scaling relations in eqs. (16), the RG estimates of ζ and z and the exact values of γ and ν, we obtain $\tau = 5/4 = 1.25$ and $\alpha = 10/7 \approx 1.43$. These values can be compared with Barkhausen measurements in magnetostrictive materials [1,4,5]. Earlier measurements by Urbach *et al*

[1] gives $\tau = 1.33 \pm 0.10$. More recently, Durin and Zapperi (DZ) [5] reported the more accurate exponents $\tau = 1.28 \pm 0.02$ and $\alpha = 1.5 \pm 0.1$, which are close to our estimates. In the last case the experiments were performed under an applied stress. Positive magnetostrictive materials show an increase in internal domain wall lengths under applied stress, which make possible the analysis of finite size effects in experimental results. According to DZ measurements, the signal amplitude cutoff increases with the stress while the avalanche duration cutoff decreases with stress, in such a way that the avalanche size cutoff s_c is nearly independent on stress. The stress dependence of the signal amplitude cutoff was also observed in [4]. We still do not understand the scaling of the distribution cutoffs with applied stress. However, the absence of stress dependence in s_c suggests that the system is in a subcritical state, where the correlation length is smaller than system size. Unfortunately, the demagnetization factor is kept constant in these experiments and, therefore, one can not determine if it is actually a control parameter.

On the other hand, numerical simulations of the cellular automaton version of eq. (1) have also been performed [4,5]. The more accurate numerical estimates, reported by Durin and Zapperi [5], are $\tau = 1.26 \pm 0.04$ and $\alpha = 1.40 \pm 0.05$, which are in very good agreement with our estimates. However, as in experiments, the demagnetization factor is kept constant. Numerical simulations of the same model, but including long-range dipolar interactions, clearly shows that the demagnetization factor is a control parameter [3]. We expect the same result when long-range dipolar interactions are absent.

Hence, we strongly suggest to change the demagnetization factor in both, experiments and simulations. In numerical simulations it is reduced to change a model parameter, while it can be done in experiments changing, for instance, sample geometry. Changing the demagnetization factor will be a very good test to our model predictions. After that, one may analyze if the avalanche distribution cutoffs are a consequence of finite size effects or just an evidence that the system is in a subcritical state. Based on numerical simulations, Bahiana *et al* [4] rule out the second possibility. However, they may have taken the demagnetization factor so small that the correlation length is smaller than system size. Decreasing the demagnetization factor the system will, according to our predictions, evolve to the subcritical state.

We conclude that the long-range demagnetization field does not change the universality class from that of local demagnetization field. This result contradicts previous conclusion by Urbach *et al* [1], based on numerical simulations in one and two dimensions and Barkhausen noise measurements. According to them, the addition of a long-range demagnetization field results in self-organized criticality, while a local demagnetization field does not. However, in our opinion, their analysis only reveals that long-range demagnetization effects are more appropriate to describe the domain wall motion in magnetic materials. As in other numerical simulations and experiments, the demagnetization factor is kept constant.

This work has been partly supported by the *Alma Mater* prize , from the Havana University, and Ministerio de Educación y Cultura, Spain.

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